Similarities between encryption and decryption: how far can we go?

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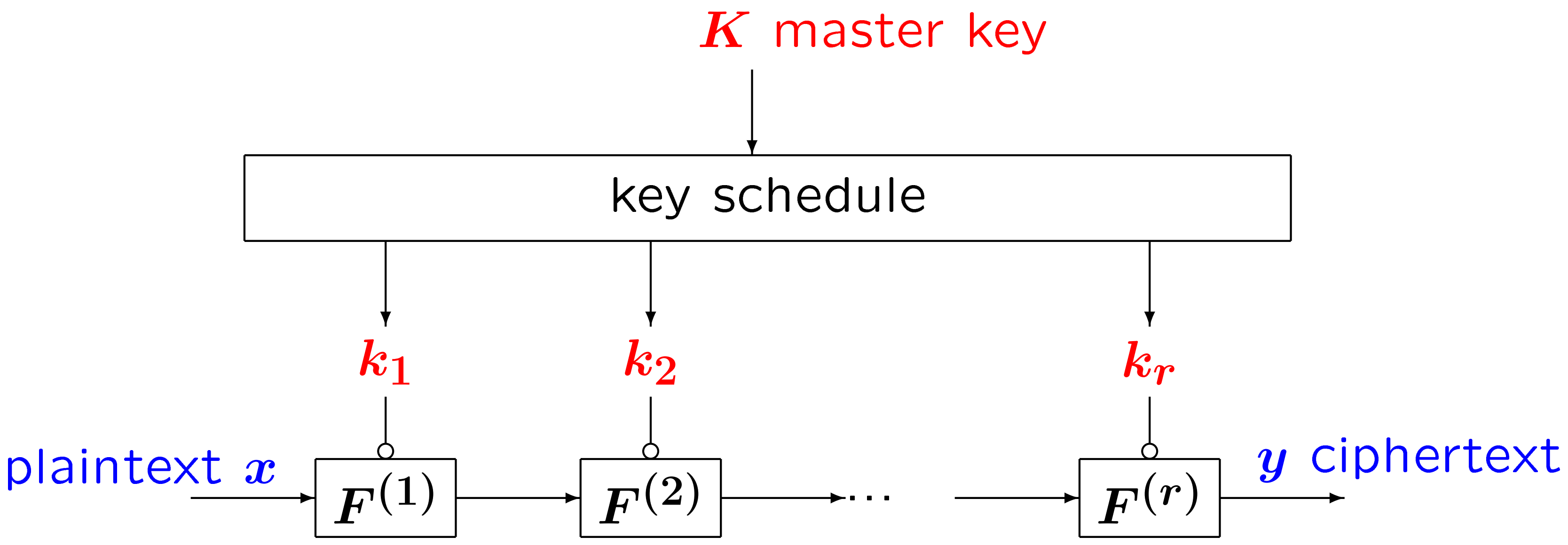
Outline

- Low-latency and lightweight ciphers
- Minimizing the overhead of decryption: involutional ciphers and involutional building-blocks
- Minimizing the overhead of decryption: reflection ciphers
- PRINCE

Iterated block ciphers



where each $F^{(i)}$ is a keyed permutation of $\mathbf{F}_2^n$.

Lightweight block ciphers

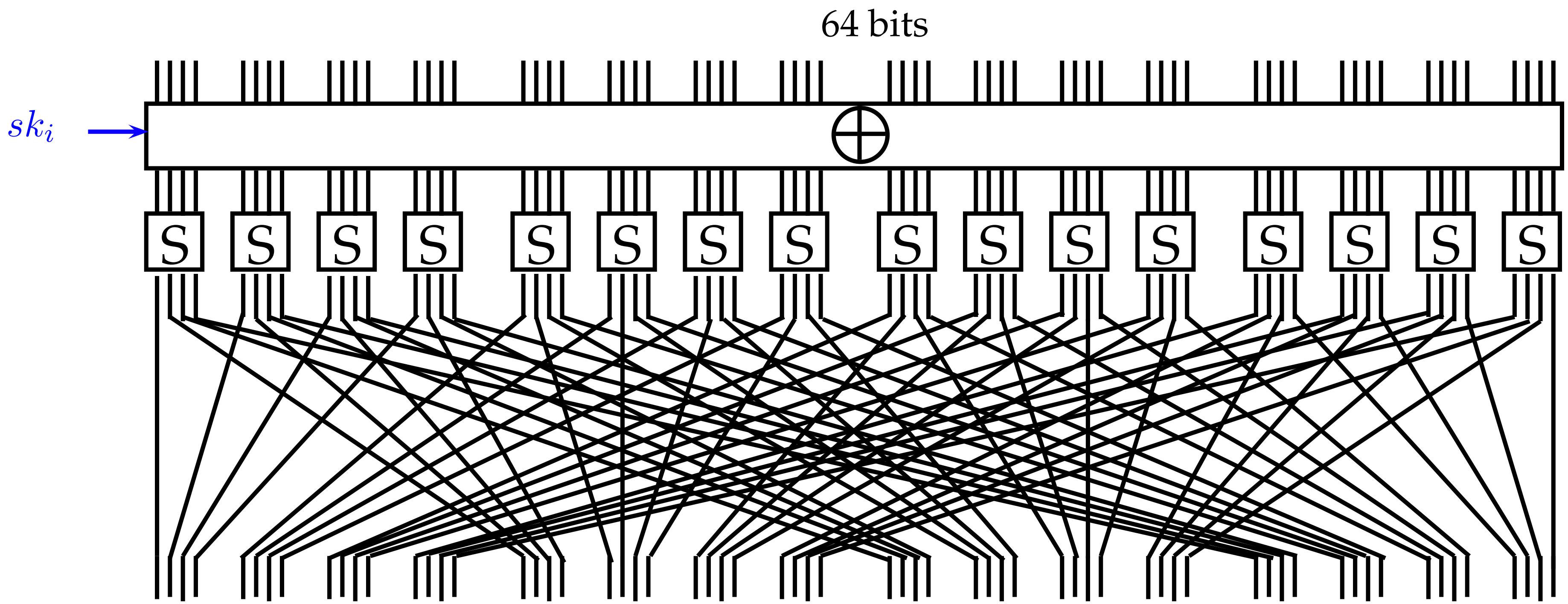
AES [Daemen-Rijmen 98][FIPS PUB 197]

- blocksize: $\mathbf{128}$ bits
- Sbox operates on $\mathbf{8}$ bits
- linear diffusion layer is a linear permutation of $\left(\mathbf{F}_{\mathbf{2}^8}\right)^{\mathbf{4}}$

To make it smaller in hardware:

- blocksize: $\mathbf{64}$ bits
- smaller Sbox, on $\mathbf{3}$ or $\mathbf{4}$ bits
- linear diffusion layer over a smaller alphabet
- simplified key-schedule

The usual design strategy: PRESENT [Bogdanov et al. 07]



31 rounds (+ a key addition)

Lightweight but secure...

Increase the number of rounds!

- PRESENT [Bogdanov et al. 07]. 31 rounds
- LED [Guo et al. 11]:
  LED-64: 32 rounds, LED-128: 48 rounds
- SPECK [Beaulieu et al. 13]:
  SPECK64/128: 27 rounds, SPECK128/256: 34 rounds
- SIMON [Beaulieu et al. 13]:
  SIMON64/128: 44 rounds, SIMON128/256: 72 rounds

Does lightweight mean "light + wait"? [Knežević et al. 12]

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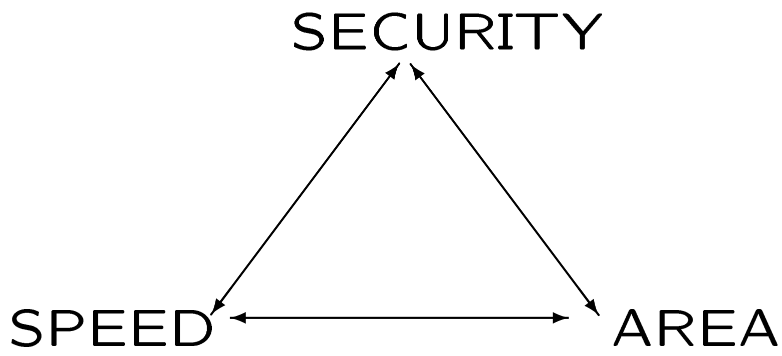


Low-latency encryption.

- Memory encryption
- VANET (Vehicular ad-hoc network)
- encryption for high-speed networking...

How can we design a fast and lightweight cipher?

Unrolled implementation.

- small number of rounds;
- each round of encryption and decryption should have a low implementation cost;
- the rounds do not need to be similar.

Related open problem.

Is it possible to provide security arguments for a cipher iterating very different rounds?

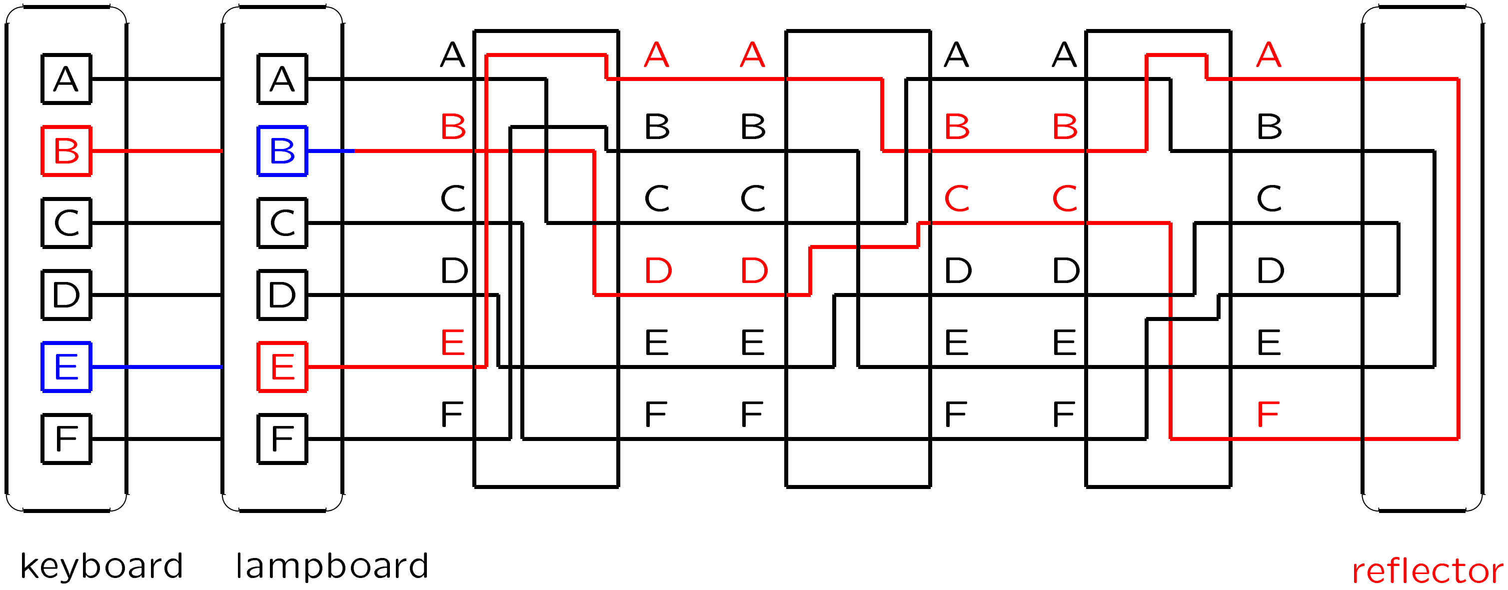
Minimizing the overhead of decryption:

involutional building-blocks

When lightweight encryption was really an issue...

`http://www.nsa.gov/museum/enigma.html`

Scherbius' solution: add a reflector



$$E_K = F_K^{-1} \circ M \circ F_K \text{ where } M = M^{-1}$$

Can E_K be an involution?

Fixed points. [Youssef-Tavares-Heys 96]

- A random permutation of $\mathbf{F}_2^n$ has 1 fixed point on average;
- A random involution of $\mathbf{F}_2^n$ has $2^{\frac{n}{2}} + \mathcal{O}(1)$ fixed points.

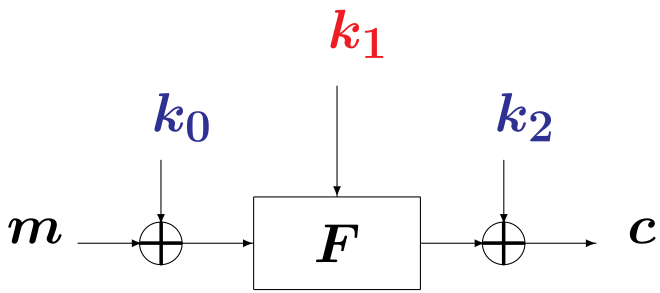
In particular, for $E_K = F_K^{-1} \circ M \circ F_K$

E_K has the same cycle structure (and the same number of fixed points) as M.

- Enigma: the reflector has no fixed points;
- DES with a weak key: M is the swapping of the 2 halves
  $\rightarrow$ It has 2^{32} fixed points [Coppersmith 85].

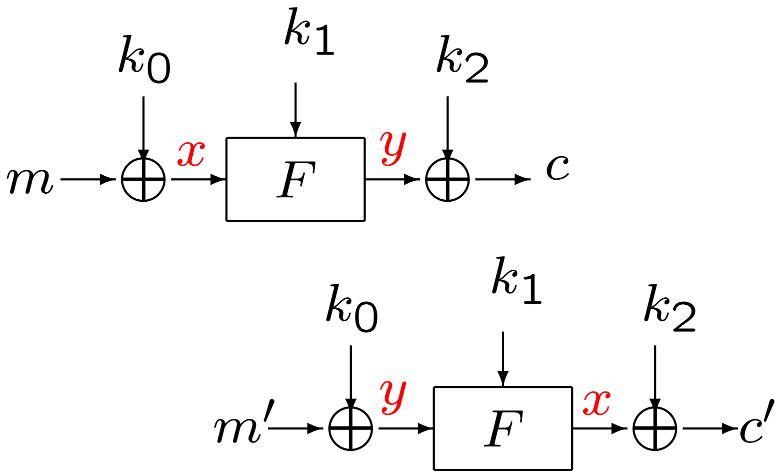
Add some whitening keys [Rivest 84]

FX construction



Slide attack with complexity $2^{\frac{n+1}{2}}$

[Youssef-Tavares-Heys 96][Dunkelman et al. 12]



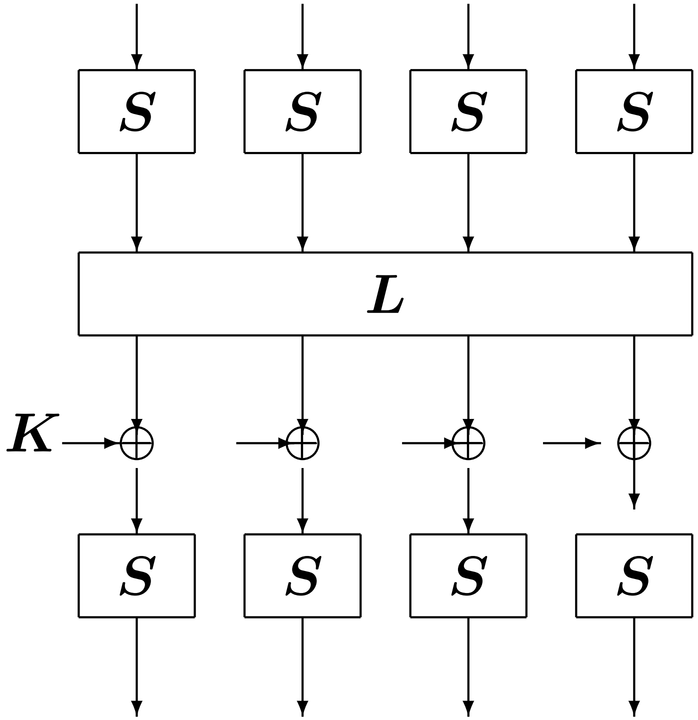
If (m, c) and (m', c') satisfy $m \oplus c = m' \oplus c'$,
then check whether $k_0 \oplus k_2 = m' \oplus c$.

Using involutional building-blocks

Examples:

- Feistel ciphers
- involutional SPNs [Youssef-Tavares-Heys 96]
- Khazad [Barreto-Rijmen 00]
- ANUBIS [Barreto-Rijmen 00]
- NOEKEON [Daemen et al. 00]
- ICEBERG [Standaert et al. 04]...

AES superbox



$\boldsymbol{S}$ is a permutation over $\mathbf{F}_2^m$

The diffusion layer is linear over $\mathbf{F}_{2^m}$ and has maximal branch number.

Involutional Sboxes with an SPN

Maximal expected probability for a two-round differential:

$$\mathrm{MEDP}_2 = \max_{a \neq 0, b} \Pr_{x,K}[\Delta E_K(x) = b | \Delta x = a]$$

For the AES Sbox $S(x) = \ell(x^{254})$:

$\mathrm{MEDP}_2 = 53 \times 2^{-34}$ [Keliher-Sui 07]

For the naive Sbox $S(x) = x^{254}$:

$\mathrm{MEDP}_2 = 79 \times 2^{-34}$ [Daemen-Rijmen 06]

$\rightarrow$ Highest possible value for a function having similar values in its difference table [Park et al. 03]

A new bound (particular case) [C.-Roué 13]

Consider an SPN with a nonlinear layer composed of t parallel applications of a function S over $\mathbf{F}_{2^m}$ and with an MDS linear diffusion layer over $\mathbf{F}_{2^m}$, if $S(x) = \ell(x^s)$ or $S(x) = (\ell(x))^s$ where ℓ is an affine permutation of $\mathbf{F}_2^m$, we have

$$\mathrm{MEDP}_2 \leq 2^{-m(t+1)} \max_{1 \leq u \leq t} \max_{\alpha,\beta \neq 0} \sum_{\gamma \in \mathbf{F}_{2^m}^*} \delta(\alpha,\gamma)^u \delta(\gamma,\beta)^{t+1-u}$$

where $\delta(a,b) = \#\{x \in \mathbf{F}_2^m, \;\; S(x+a) + S(x) = b\}$.

Moreover, the bound is tight for all MDS linear layers if one of the following conditions holds:

- $S(x) = x^s$;
- $S(x) = \ell(x^s)$ and the maximum is attained for $u = 1$.

Difference table of the inverse function over $\mathbb{F}_{16}$

	1	ζ	ζ^2	ζ^3	ζ^4	ζ^5	ζ^6	ζ^7	ζ^8	ζ^9	ζ^{10}	ζ^{11}	ζ^{12}	ζ^{13}	ζ^{14}
1	4	0	0	0	0	2	0	2	0	0	2	2	0	2	2
ζ	0	0	0	0	2	0	2	0	0	2	2	0	2	2	4
ζ^2	0	0	0	2	0	2	0	0	2	2	0	2	2	4	0
ζ^3	0	0	2	0	2	0	0	2	2	0	2	2	4	0	0
ζ^4	0	2	0	2	0	0	2	2	0	2	2	4	0	0	0
ζ^5	2	0	2	0	0	2	2	0	2	2	4	0	0	0	0
ζ^6	0	2	0	0	2	2	0	2	2	4	0	0	0	0	2
ζ^7	2	0	0	2	2	0	2	2	4	0	0	0	0	2	0
ζ^8	0	0	2	2	0	2	2	4	0	0	0	0	2	0	2
ζ^9	0	2	2	0	2	2	4	0	0	0	0	2	0	2	0
ζ^{10}	2	2	0	2	2	4	0	0	0	0	2	0	2	0	0
ζ^{11}	2	0	2	2	4	0	0	0	0	2	0	2	0	0	2
ζ^{12}	0	2	2	4	0	0	0	0	2	0	2	0	0	2	2
ζ^{13}	2	2	4	0	0	0	0	2	0	2	0	0	2	2	0
ζ^{14}	2	4	0	0	0	0	2	0	2	0	0	2	2	0	2

MEDP_2 for AES and variants

$$2^{-m(t+1)} \max_{1 \le u \le t} \max_{\alpha,\beta \ne 0} \sum_{\gamma \in \mathbf{F}_{2^m}^*} \delta(\alpha,\gamma)^u \delta(\gamma,\beta)^{t+1-u}$$

AES Sbox $S(x) = \ell(x^{254})$**.**

$$\longrightarrow \mathrm{MEDP}_2 = 53 \times 2^{-34}$$

Naive Sbox $S(x) = x^{254}$**.**

$$\delta(a,b) = \delta(b,a)$$

$$\begin{aligned} \max_{\alpha,\beta \ne 0} \sum_{\gamma \in \mathbf{F}_{2^m}^*} \delta(\alpha,\gamma)^u \delta(\gamma,\beta)^{t+1-u} &= \max_{\alpha,\beta \ne 0} \sum_{\gamma \in \mathbf{F}_{2^m}^*} \delta(\alpha,\gamma)^u \delta(\beta,\gamma)^{t+1-u} \\ &= \max_{\alpha \ne 0} \sum_{\gamma \in \mathbf{F}_{2^m}^*} \delta(\alpha,\gamma)^{t+1} \end{aligned}$$

$$\longrightarrow \mathrm{MEDP}_2 = 79 \times 2^{-34}$$

Minimizing the overhead of decryption:

reflection ciphers

Reflection ciphers

Definition. A block cipher E is a reflection cipher if there exists a permutation P of the key space such that, for all K,

$$(E_K)^{-1} = E_{P(K)}$$

Examples.

- Feistel cipher with independent round keys:

$$P(k_1, \ldots, k_r) = (k_r, \ldots, k_1)$$

- RSA:

$$P = \text{inversion modulo } (p-1)(q-1).$$

Properties of the coupling permutation

$$(E_K)^{-1} = E_{P(K)}$$

implies

$$E_K = E_{P^2(K)}$$

Choice of P.

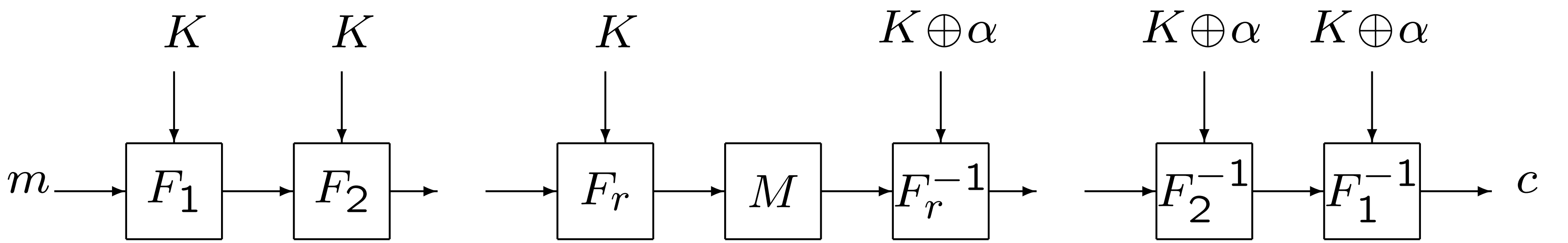
P should be an involution.

Example:

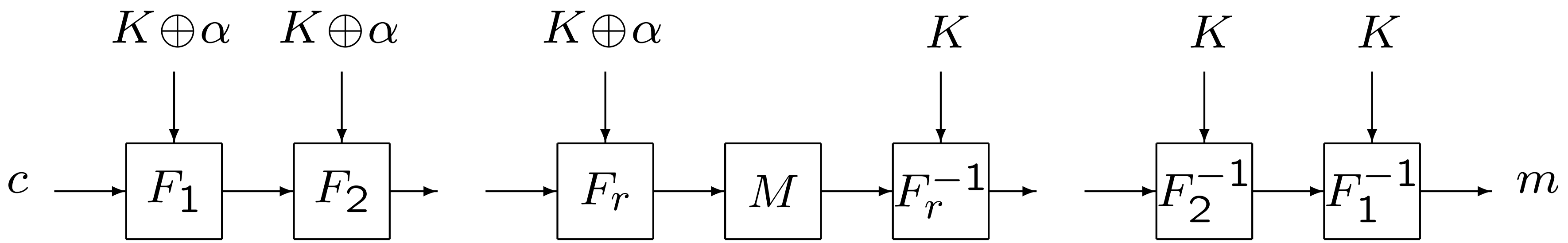
$$P(K) = K \oplus \alpha$$

Iterated reflection cipher with $P(K) = K \oplus \alpha$

Encryption:

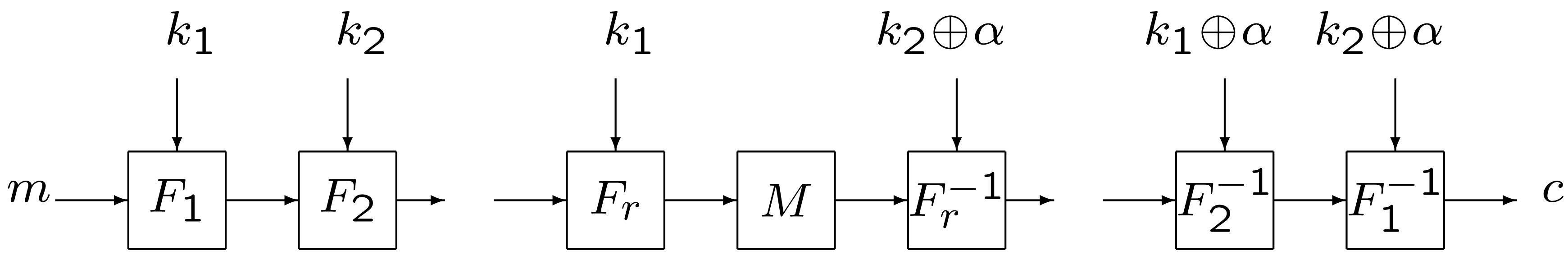


Decryption:



where M is an involution.

Example of a reflection cipher with $P(k_1, k_2) = (k_2 \oplus \alpha, k_1 \oplus \alpha)$



$$\left(\boldsymbol{E}_{(\boldsymbol{k_1},\boldsymbol{k_2})}\right)^{-1} = \boldsymbol{E}_{(\boldsymbol{k_2}\oplus\boldsymbol{\alpha},\boldsymbol{k_1}\oplus\boldsymbol{\alpha})}$$

For all keys with $\boldsymbol{k_2 = k_1 \oplus \alpha}$, the cipher is an involution,
and it has the same number of fixed points as $\boldsymbol{M}$.
$\longrightarrow$ Large class of weak keys.

Fixed points of the coupling permutation

Fixed points of P.

The keys for which the encryption function is an involution can be detected with $\mathcal{O}(2^{\frac{n}{2}})$ plaintext-ciphertext pairs.

Choice of P.

P should be an involution without fixed points.

Example:

$$P(K) = K \oplus \alpha$$

On related-key distinguishers for reflection ciphers

Trivial related-key distinguishers:

are not considered.

(they may be important in some scenarios, e.g., [Iwata-Kurosawa 03])

Related-key distinguishers:

may have an impact in a single-key model.

A related-key distinguisher for E_K involving two keys K and K' related by $K' = P(K)$ is a distinguisher in the single-key model.

$\longrightarrow$ Related-key distinguishers may be relevant!

On differential related-key distinguishers

Distinguishers involving K and $K' = P(K)$ should be avoided.

Two strategies:

- Choose P such that the existence of such distinguishers is very unlikely, e.g., such that $K \oplus P(K)$ has always a high weight;
- Choose P such that such related-key distinguishers can be exploited for a few K only.

Trade-off between

$$\min_K wt(K \oplus P(K)) \text{ and } \max_\delta \#\{K : K \oplus P(K) = \delta\}$$

For $P(K) = K \oplus \alpha$ where $wt(\alpha)$ is high, we maximize the first quantity.

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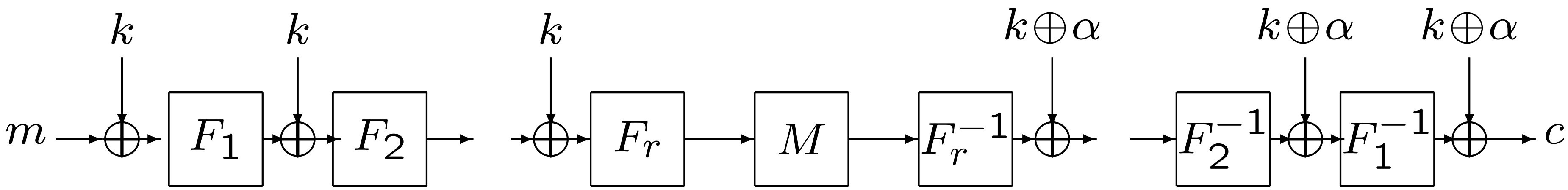
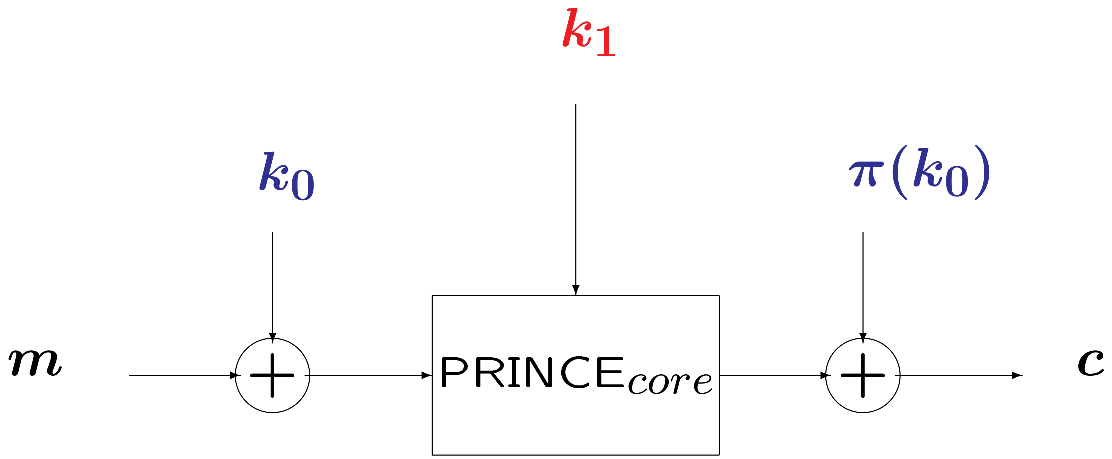
Reflection cipher with $P(K) = K \oplus \alpha$



Increasing the key length

FX construction [Rivest 84]

$$\boldsymbol{k} = (\boldsymbol{k_0} || \boldsymbol{k_1})$$

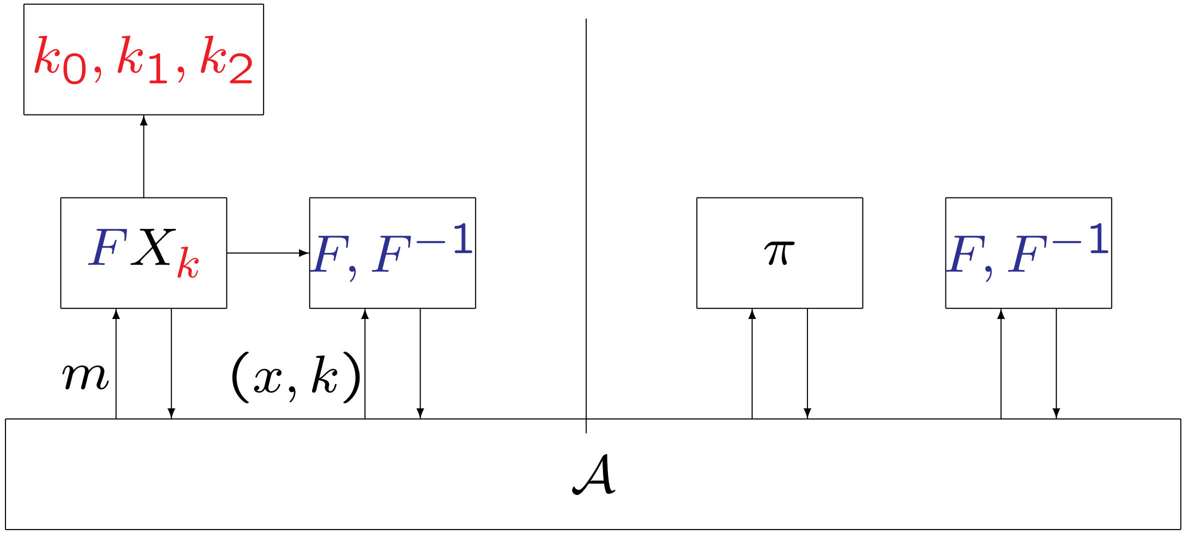


with $\boldsymbol{\pi(x) = (x \ggg 1) \oplus (x \gg 63)}$

$\rightarrow$ $(\boldsymbol{k_0 \oplus k_1, \pi(k_0) \oplus k_1})$ takes all possible values when $(\boldsymbol{k_0, k_1})$ varies.

Security of the FX construction [Kilian-Rogaway 96]

$$FX_{k_0,k_1,k_2}(m) = F_{k_1}(m \oplus k_0) \oplus k_2$$



The advantage of any adversary who makes D queries to $E = FX$ and T queries to (F, F^{-1}) is at most

$$DT2^{-(\kappa_1+n-1)}$$

Impact of the reflection property on the FX construction

Ideal reflection cipher with coupling permutation P.

If $\boldsymbol{P}$ is an involution without fixed points, the key space can be decomposed as

$$\mathbb{F}_2^{\kappa_1} = \boldsymbol{H} \cup \boldsymbol{P}(\boldsymbol{H})$$

where $\boldsymbol{H}$ contains half of the keys.

Let $\boldsymbol{F}$ be an ideal block cipher with key space $\boldsymbol{H}$.

We extend it by

$$\tilde{\boldsymbol{F}}_{\boldsymbol{k}}(\boldsymbol{x}) = \begin{cases} \boldsymbol{F}_{\boldsymbol{k}}(\boldsymbol{x}) & \text{if } \boldsymbol{k} \in \boldsymbol{H} \\ \boldsymbol{F}^{-1}_{\boldsymbol{P}(\boldsymbol{k})}(\boldsymbol{x}) & \text{if } \boldsymbol{k} \in \boldsymbol{P}(\boldsymbol{H}) \end{cases}$$

Security of the $\tilde{F}X$ construction.

The advantage of any adversary who makes $\boldsymbol{D}$ queries to $\boldsymbol{E} = \tilde{\boldsymbol{F}}\boldsymbol{X}$ and $\boldsymbol{T}$ queries to $(\boldsymbol{F}, \boldsymbol{F}^{-1})$ is at most

$$\boldsymbol{DT}2^{-(\kappa_1+n-2)}\ .$$

Parameters

- Block size: 64 bits
- Key size: 128 bits
- Nb of Sbox layers: 12

Security claim in the single-key model:

126-bit security

There is no attack with time and data complexities are such that

$$DT \ll 2^{126}$$

Best attack.
MitM attack on 8 rounds with $DT = 2^{124}$
[C. Naya-Plasencia Vayssière 13].

Conclusions and open issues

- Involutional building-blocks may introduce some weaknesses in some cases. How can we use them in secure way?

- Reflection ciphers considerably reduce the overhead on decryption on top of encryption for unrolled implementations.

- Find some other key schedules (work in progress).